

The new compaction equation in [Densification paper](#) was described as follows:

Below a thin surface layer of new snow, in which there is rapid metamorphism and settling, and for  $\rho \leq 550 \text{ kg m}^{-3}$ , we write

$$\dot{\epsilon} = \frac{k_0^*}{\rho_w g} \left( \frac{\rho_i - \rho}{\rho} \right) (1 - \bar{M}_0 m) \frac{1}{H(\tau)} \exp\left(-\frac{E_\alpha}{RT}\right) \sigma \quad (1)$$

where  $H(\tau)$  is a ‘‘temperature-history function’’ defined by

$$H(\tau) = \int_{\tau_0}^{\tau} \exp\left(-\frac{E_\alpha}{RT(\tau')}\right) d\tau'. \quad (2)$$

Here  $\rho_i$  and  $\rho_w$  are the densities of ice and water,  $R$  is the gas constant,  $E_\alpha$  is the apparent activation energy for compaction at temperature  $T$  and  $k_0^*$  is the densification constant. The time from deposition is  $\tau$ , and the time at which the snow element leaves the thin surface layer is  $\tau_0$ .

This equation represents a simple conceptual model based on one densification process, with activation energy  $E_\alpha$ . The strength of the snow decreases as current temperature increases, but increases with past exposure to higher temperatures. That is, we suppose that the rate-limiting process in developing strength is the same process that moves grains closer together, because it allows new bonds to form. One consequence of this choice is that the strength is related to the current density; the other is that the effect of exposure to temperature on the strength can be expressed by the temperature history function  $H(\tau)$ .

**This description was based on an incorrect calculation of the history function. The corrected text describing this calculation reads as follows:**

Using equation A1 to calculate  $T(\tau)$  we find that the history function,  $H(\tau)$ , is virtually linear below the first annual layer, with a gradient that decreases with increasing  $\bar{a}$ . Figure 1(c) shows the history function for a typical activation energy  $E_\alpha = 110 \text{ kJ mol}^{-1}$  at T41D. (The determination of  $E_\alpha$  is discussed in detail in Section 6.3). The gradient of the linear fit to  $H(\tau)$  is  $1.4 \cdot 10^{-23} \text{ a}^{-1}$ . Varying the accumulation over the range 0.02 – 0.6 m w.e.  $\text{a}^{-1}$  at fixed mean annual temperature produces a variation in gradient of  $(4.8 - 0.6) \cdot 10^{-23} \text{ a}^{-1}$ .

We now make the approximation

$$\begin{aligned} H(\tau) &\approx \exp\left(-\frac{E_H}{RT_m}\right) \tau \\ &\approx -\exp\left(-\frac{E_H}{RT_m}\right) \frac{\bar{\sigma}}{\bar{a}\rho_w g} \end{aligned} \quad (3)$$

where  $E_H$  is an effective activation energy which will be close, but not equal, to  $E_\alpha$ , and  $\bar{\sigma}$ , the time-invariant average stress at a given depth, is defined as  $-\bar{a}g\rho_w\tau$ . Equation (1) then becomes

$$\dot{\epsilon} \approx -\bar{a}k_0^* \left( \frac{\rho_i - \rho}{\rho} \right) (1 - \bar{M}_0 m) \exp\left(\frac{E_H}{RT_m}\right) \exp\left(-\frac{E_\alpha}{RT}\right) \frac{\sigma}{\bar{\sigma}} \quad (4)$$

For the deeper parts of the snowcover, where  $\sigma \approx \bar{\sigma}$  and  $T \rightarrow T_m$ , equation (4) reduces to

$$\dot{\epsilon} \approx -\bar{a}k_0^* \left( \frac{\rho_i - \rho}{\rho} \right) (1 - \bar{M}_0 m) \exp\left(-\frac{(E_\alpha - E_H)}{RT_m}\right) \quad (5)$$

and there is no trend in the strain rate with increasing overburden. For  $\rho = \rho_0$ ,  $m = 0$  and equation (5) reduces further to

$$\dot{\epsilon} \approx -\bar{a}k_0^* \left( \frac{\rho_i - \rho}{\rho} \right) \exp\left(-\frac{(E_\alpha - E_H)}{RT_m}\right) \quad (6)$$

which has the same form as the purely empirical equation derived by *Herron and Langway* (1980) for snow with  $\rho \leq 550 \text{ kg m}^{-3}$

$$\dot{\epsilon} = -\bar{a}k_0^* \left( \frac{\rho_i - \rho}{\rho} \right) \exp\left(-\frac{E^*}{RT_m}\right). \quad (7)$$

The *Herron and Langway* (1980) data give  $k_0^* = 11 (+6, -4) \text{ m w.e.}^{-1}$  and  $E^* = 10.16 \pm 0.94 \text{ kJ mol}^{-1}$ .

**The conclusion that this is close to the range of values of  $E_\alpha - E_H$  shown in Table 1 for three polar locations with widely-differing climate conditions is now only true for  $E_\alpha = 200 \text{ kJ mol}^{-1}$ . Since the best value for the EGIG line data is in fact  $112 \text{ kJ mol}^{-1}$  it looks as if the hypothesis that there is only one process cannot hold. Another term in  $T_m$  is**

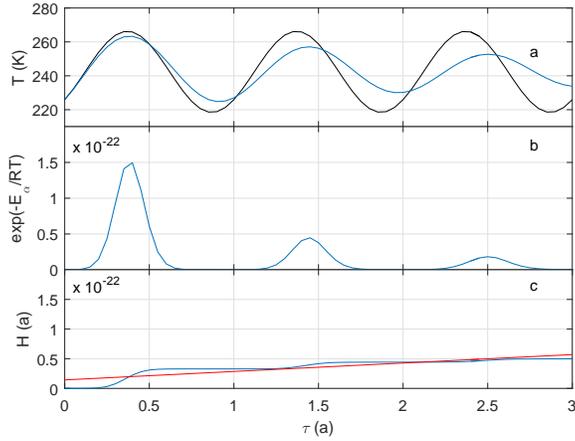


Figure 1: Estimated effect of the annual temperature variation at T41D where  $\bar{a} = 0.22$  m w.e.  $\text{a}^{-1}$  and  $T_m = 242.34$  K ( $T_m^* = -30.8^\circ\text{C}$ ). (a) The temperature,  $T$ , at the surface (black) and for an element of snow deposited at time  $\tau = 0$  as it moves downwards in the pack (blue). As the snow ages the amplitude of the temperature wave it experiences decreases. (b) The magnitude of the rate factor for a temperature-driven process with activation energy  $E_\alpha = 110$   $\text{kJ mol}^{-1}$ . This decreases more rapidly with time but is still significant for several years. (c) The temperature-history function  $H(\tau)$  given  $E_\alpha = 110$   $\text{kJ mol}^{-1}$  (blue) and a linear fit (red).

Table 1:  $(E_\alpha - E_H)$  in  $\text{kJ mol}^{-1}$ .

Site	$T_m^*$ $^\circ\text{C}$	$\bar{a}$ m w.e. $\text{a}^{-1}$	$E_\alpha = 60$ $\text{kJ mol}^{-1}$	$E_\alpha = 110$ $\text{kJ mol}^{-1}$	$E_\alpha = 200$ $\text{kJ mol}^{-1}$
South Pole	-51.0	0.07	2.7	6.5	13.9
T41D	-30.8	0.22	1.5	4.0	9.7
Roi Baudouin	-15.0	0.38	0.8	2.6	7.1

required. The easiest way to make the new equation correspond to *Herron and Langway (1980)* is by substituting  $k_0$  for  $k_0^*$  in equation 1 where

$$k_0 = k_0^* \exp\left(-\frac{E^* - (E_\alpha - E_H)}{RT_m}\right) \quad (8)$$

Note that there is also an error in section 5.4 where the wrong value of  $E_\alpha - E_H$  is used to calculate  $k_0^*$ . The lower curve in Figure 15 is not now relevant

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